

# Tucker Decomposition For Rotated Codebook in 3D MIMO System Under Spatially Correlated Channel

Fang Yuan

## Abstract

This correspondence proposes a new rotated codebook for three-dimensional (3D) multi-input-multi-output (MIMO) system under spatially correlated channel. To avoid the problem of high dimensionality led by large antenna array, the rotation matrix in the rotated codebook is proposed to be decomposed by Tucker decomposition into three low-dimensional units, i.e., statistical channel direction information in horizontal and vertical directions respectively, and statistical channel power in the joint horizontal and vertical direction. A closed-form suboptimal solution is provided to reduce the computational complexity in Tucker decomposition. The proposed codebook has a significant dimension reduction from conventional rotated codebooks, and is applicable for 3D MIMO system with arbitrary form of antenna array. Simulation results demonstrate that the proposed codebook works very well for various 3D MIMO systems.

## I. INTRODUCTION

Three-dimensional (3D) multi-input-and-multi-output (MIMO) systems are promising to meet the ever-growing data demand in future 5th Generation (5G) cellular networks, where large antenna arrays are equipped at the base station (BS) to employ 3D beamforming to serve users [1, 2].

The promised performance of 3D MIMO system largely depends on the accuracy of the channel direction information (CDI) obtained at the BS. In time division duplexing (TDD) system, the CDI can be obtained by channel estimation in uplink, but may be contaminated by pilot reuse [3]. In frequency division duplexing (FDD) system, limited feedback is widely used, where the CDI is quantized at the user and then fed back to the BS [4]. Yet the feedback overhead is considered unacceptable under large antenna array [5].

Recently, the works in [6, 7] have revealed that the spatial correlation can be exploited to reduce the feedback overhead significantly for 3D MIMO system with large antenna array. The spatial correlation is observed very typical in MIMO channels [8], due to the small antenna spacing in the array and low angular spread in the propagation. It indicates that FDD is also applicable for large antenna array system.

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Fang Yuan is with Smart Wireless Laboratory, National Institute of Information and Communications Technology (NICT), Japan (email: [yuanfang@nict.go.jp](mailto:yuanfang@nict.go.jp)).

In limited feedback, various codebooks have been proposed for spatially correlated channels. The Lloyd-like codebooks [9, 10] have good quantization performance, but cannot be used off-line. Discrete Fourier transformation (DFT) codebook is proposed for highly correlated channels with uniform linear array (ULA) at the BS [11], which has desirable features such as constant modulus and finite alphabet. For 3D MIMO system with uniform rectangular array (URA), a Kronecker-product DFT codebook was proposed in [12]. However, as shown in [11], the performances of these pure DFT based codebooks degrade severely when the angular spreads in the channel increase.

A well-known codebook, rotated codebook, has been proposed in [13], which transforms the codewords optimized for uncorrelated channels (e.g., Grassmannian linear packing (GLP) codewords) by channel correlation matrix. The rotated codebook is extended for multiuser MIMO system in [14]. It was proved that the rotated codebook is asymptotically optimal in quantizing any spatially correlated channels as the codebook size becomes large [15]. Therefore, the rotated codebook serves as a performance upper bound for other books in practice and widely applied. Theoretically, the rotated codebook can be applied directly in 3D MIMO system with large antenna array. However, the high dimensionality in the correlation matrix incurs not only heavy feedback load for statistical information, but also high computation complexity in the matrix operations [16]. Moreover, it is challenging to acquire an accurate correlation matrix required by the rotated codebook under high dimensionality, which is known as the “curse of the dimensionality” [17].

Many research efforts have been made for the dimension reduction in the limited feedback. For example, the singular value decomposition (SVD) is employed to alleviate the dimension problem by discarding non-dominant eigenvectors in the correlation matrix [18]. Moreover, inspired by 3D MIMO system with URA, independent quantization is considered, which naively quantizes the CDI of 3D MIMO in horizontal and vertical directions independently and reuses existing codebooks in each direction [19]. Independent quantization avoids the problem of high dimensionality led by large antenna array, while the performance degrades as the angular spreads become large.

In this correspondence, we propose to apply the Tucker decomposition in the rotation matrix required by the rotated codebook, which aims at solving the problem of high dimensionality led by large antenna array in 3D MIMO system. A closed-form solution is provided for the Tucker decomposition, which is suboptimal but with low computational complexity. Simulation results demonstrate that the proposed codebook yields good performance for 3D MIMO systems with different antenna arrays.

*Notations:*  $(\cdot)^T$ ,  $(\cdot)^H$  and  $(\cdot)^*$  are respectively the transpose, Hermitian and conjugate operation,  $\otimes$  and  $\|\cdot\|$

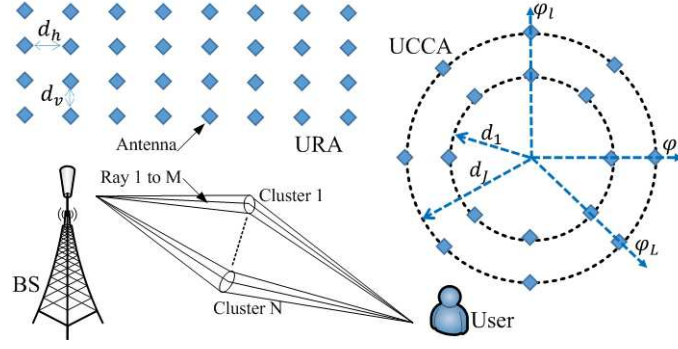


Fig. 1. An example of 3D MIMO Channel, scattering clusters and antenna arrays

denotes the Kronecker product and Frobenius norm,  $\text{diag}(\mathbf{x})$  is the diagonal matrix with diagonal entries from the vector  $\mathbf{x}$ , and  $\text{diagm}(\mathbf{X})$  is the diagonal matrix with diagonal entries the same as the matrix  $\mathbf{X}$ .

## II. SYSTEM MODEL

### A. Channel Model

Consider a downlink 3D MIMO system where the BS equipped with an array of  $N_t$  antennas [1] to serve single-antenna users in the macro cell, and all antennas are omni-directional. According to the spatial channel model in [8, 20], the 3D MIMO channel from the BS to the user consists of several scattering clusters distributed in the 3D space, as shown in Fig. 1, where in each cluster there are multiple rays with small random angle offsets. The 3D MIMO channel is expressed as a  $N_t$ -dimensional vector

$$\mathbf{h} \triangleq \sum_{n=1}^N \sum_{m=1}^M g_{n,m} \mathbf{a}_{n,m} \quad (1)$$

where  $g_{n,m}$  is the random complex gain of the  $m$ th ray in the  $n$ th cluster, and  $\mathbf{a}_{n,m}$  of dimension  $N_t \times 1$  is the corresponding array response.

The array response  $\mathbf{a}_{n,m}$  depends on the specific form of the array mounted at the BS, as shown in [20]. The array response of URA shown in Fig. 1 can be decomposed into two ULA responses respectively in horizontal and vertical directions as

$$\mathbf{a}_{n,m} = \mathbf{a}_v(\phi_{n,m}) \otimes \mathbf{a}_h(\theta_{n,m}) \quad (2)$$

with  $\mathbf{a}_h(\theta_{n,m}) = [1, e^{-jk_h \cos \theta_{n,m}}, \dots, e^{-jk_h(N_h-1) \cos \theta_{n,m}}]^T$ ,  $\mathbf{a}_v(\phi_{n,m}) = [1, e^{-jk_v \cos \phi_{n,m}}, \dots, e^{-jk_v(N_v-1) \cos \phi_{n,m}}]^T$ , where  $k_h = 2\pi \frac{d_h}{\lambda}$ ,  $k_v = 2\pi \frac{d_v}{\lambda}$ ,  $d_h$  and  $N_h$  ( $d_v$  and  $N_v$ ) are respectively the antenna spacing and the number of antennas at the URA in horizontal (vertical) direction,  $\lambda$  is the carrier wavelength,  $\cos \theta_{n,m}$  and  $\cos \phi_{n,m}$  are

the direction cosines of the  $m$ th ray in the  $n$ th cluster respectively in the horizontal and vertical directions. The array response of uniform concentric circular array (UCCA) shown in Fig. 1 is expressed as

$$\mathbf{a}_{n,m} = [\mathbf{a}(\varphi_1)^T, \dots, \mathbf{a}(\varphi_L)^T]^T \quad (3)$$

with  $\mathbf{a}(\varphi_l) = [e^{-j2\pi\frac{d_1}{\lambda}\cos(\phi_{n,m}-\varphi_l)\cos\theta_{n,m}}, \dots, e^{-j2\pi\frac{d_J}{\lambda}\cos(\phi_{n,m}-\varphi_l)\cos\theta_{n,m}}]^T$ , where  $J$  and  $L$  are respectively as the number of rings and the number of antennas equally placed on each ring in the UCCA,  $d_j$  is the radius of the  $j$ th ring,  $\varphi_l = 2l\pi/L$  is the  $l$ th radial direction,  $\theta_{n,m}$  and  $\phi_{n,m}$  are respectively the angle of the  $m$ th ray in the  $n$ th cluster in horizontal and vertical directions.

### B. Rotated Codebook

The CDI of 3D MIMO channel is defined as  $\bar{\mathbf{h}} = \mathbf{h}/\|\mathbf{h}\|$ , which is of unit-norm [4]. In the procedure of limited feedback, each user quantizes the CDI using a codebook known by the BS, and feeds the optimal codeword back to the BS for the beamforming [4]. The rotated codebook is widely applied among all the codebooks to quantize the CDI [13], and the codewords of a  $B$ -bit rotated codebook are given by

$$\mathbf{f}_i = \frac{\mathbf{R}^{\frac{1}{2}}\mathbf{c}_i}{\|\mathbf{R}^{\frac{1}{2}}\mathbf{c}_i\|}, \quad i = 1, \dots, 2^B \quad (4)$$

where the codewords  $\mathbf{c}_i$  of dimension  $N_t \times 1$  are optimized under uncorrelated channels and universal for different users, and the rotation matrix  $\mathbf{R}$  is the spatial correlation matrix in 3D MIMO channel defined as

$$\mathbf{R} = \mathbf{E}\{\mathbf{h}\mathbf{h}^H\}, \quad (5)$$

which differs from user to user,  $\mathbf{E}\{\cdot\}$  is the expectation operation, and  $\mathbf{X}^{\frac{1}{2}}$  is the square root of matrix  $\mathbf{X}$ .

By exploiting the statistical information, the rotated codewords  $\mathbf{f}_i$  in (4) improves the quantization performance from the universal codewords  $\mathbf{c}_i$  under spatially correlated channels. It has been proved in [15] that as the codebook size increases, the rotated codebook is asymptotically optimal in quantizing arbitrary spatially correlated channel. Yet under a large antenna array in 3D MIMO system, the high dimensionality problem becomes challenging. For instance, with tens or hundreds of antennas at the array, say  $N_t = 64$  and 256, the matrix  $\mathbf{R}$  of dimension  $N_t \times N_t$  has respectively overloaded 4096 and 65536 elements.

The high dimensionality in  $\mathbf{R}$  not only increases the complexity of MIMO operations, but also challenges the application of rotated codebooks. In practice, the correlation matrix  $\mathbf{R}$  can be acquired at the BS by either the feedback from the user, or the estimation from samples collected in uplink [6]. In high dimensionality,

the former experiences a huge overhead for each feedback of statistical information, and the latter may suffer from the problem of “curse of dimensionality”. As shown in [16], when the dimension of correlation matrix is comparable with the number of samples, the widely-used “sample-covariance” estimation becomes invalid. Such a problem is recognized as “curse of dimensionality”, and is far from trivial in the estimation theory. Therefore, we strive to reduce the dimensionality of the rotation matrix in the rotated codebook.

### III. TUCKER DECOMPOSITION FOR ROTATED CODEBOOK

We propose to apply the Tucker decomposition to reduce the high dimensionality for the correlation matrix in rotated codebook under spatially correlated 3D MIMO channel.

#### A. The Proposed Codeword Structure

The proposed new rotated codebook is with the codeword structure as

$$\mathbf{f}_i = \frac{\hat{\mathbf{R}}^{\frac{1}{2}} \mathbf{c}_i}{\|\hat{\mathbf{R}}^{\frac{1}{2}} \mathbf{c}_i\|}, \quad i = 1, \dots, 2^B \quad (6)$$

where the rotation matrix  $\hat{\mathbf{R}}$  is given by a designed structure as

$$\hat{\mathbf{R}} = (\mathbf{U} \otimes \mathbf{V}) \text{diag}(\boldsymbol{\lambda}) (\mathbf{U} \otimes \mathbf{V})^H \quad (7)$$

which consists of three information units: the unitary matrix  $\mathbf{V}$  is of dimension  $N_1 \times N_1$ , the unitary matrix  $\mathbf{U}$  is of dimension  $N_2 \times N_2$ , and the vector  $\boldsymbol{\lambda}$  is with nonnegative elements of dimension  $N_t \times 1$  where  $N_1 N_2 = N_t$ . Both of  $N_1$  and  $N_2$  can be small even when  $N_t$  is large, e.g.,  $N_t = 256$  and  $N_1 = N_2 = 16$ . Thus, only three low-dimensional information units need to be fed back from the user to the BS for reconstructing the rotation matrix  $\hat{\mathbf{R}}$ , which avoids the high dimensionality problem in conventional rotated codebooks. As will be revealed shortly, the structure in (7) is led by the Tucker decomposition to the correlation matrix  $\mathbf{R}$ .

#### B. Target of Each Information Unit

The three information units in the new rotation matrix can target at different information in the CDI of 3D MIMO channel. To see this, we take the 3D MIMO channel with URA at the BS as an example.

Mathematically, given  $N_t = N_1 N_2$ , any channel vector  $\mathbf{h}$  of dimension  $N_t \times 1$  can be reshaped into a matrix form as  $\mathbf{H}$  of dimension  $N_1 \times N_2$ , e.g., via the “reshape” function provided in Matlab, and consequently

$$\mathbf{H} = \text{reshape}\{\mathbf{h}\} \text{ and } \mathbf{h} = \text{vec}\{\mathbf{H}\} \quad (8)$$

where  $\text{vec}(\cdot)$  means vectorizing a matrix into a vector. By setting  $N_1 = N_h$  and  $N_2 = N_v$  for URA, it has  $\mathbf{H} = \sum_{n=1}^N \sum_{m=1}^M g_{n,m} \mathbf{a}_h(\theta_{n,m}) \mathbf{a}_v(\phi_{n,m})^T$ , where the columns and rows of  $\mathbf{H}$  stand for the channel information in horizontal and vertical directions respectively.

The information units  $\mathbf{V}$  and  $\mathbf{U}$  can target at statistical direction information respectively in horizontal and vertical directions, through the SVD of the left and right correlation matrices of 3D MIMO channel  $\mathbf{H}$  as

$$\mathbf{R}_h = \mathbf{E}\{\mathbf{H}\mathbf{H}^H\} = \mathbf{V} \text{diagv}(\boldsymbol{\lambda}_h) \mathbf{V}^H \text{ and } \mathbf{R}_v = \mathbf{E}\{\mathbf{H}^T \mathbf{H}^*\} = \mathbf{U} \text{diagv}(\boldsymbol{\lambda}_v) \mathbf{U}^H \quad (9)$$

Let  $\mathbf{G} = \mathbf{V}^H \mathbf{H} \mathbf{U}^*$  be the instantaneous channel gain of  $\mathbf{H}$  projected onto the two unitary matrices of  $\mathbf{V}$  and  $\mathbf{U}$ , and  $\mathbf{R}_g = \mathbf{E}\{\text{vec}\{\mathbf{G}\} \text{vec}\{\mathbf{G}\}^H\}$  be the corresponding correlation matrix with the diagonal as the statistical channel power. Then, the information unit  $\boldsymbol{\lambda}$  can target at the statistical channel power in the joint horizontal and vertical direction by satisfying

$$\text{diagv}(\boldsymbol{\lambda}) = \text{diagm}(\mathbf{R}_g) \quad (10)$$

In other words, the information unit  $\boldsymbol{\lambda}$  characterizes the interaction between the information unit  $\mathbf{V}$  and  $\mathbf{U}$ .

So far, the physical explanations for the target of each information unit is given for URA. Actually, the explanations for horizontal and vertical directions under URA are similar to that between transmit and receive ends under MIMO channel studied in [21] and [22]. It should be noted that the equations in (8), (9) and (10) hold for arbitrary antenna array, not only for URA. Realizing this, the idea of decomposing the channel information in horizontal and vertical directions under URA can be *generalized* for 3D MIMO channel under arbitrary antenna array when (8), (9) and (10) is applied. However, unlike under URA, physical explanations for (8), (9) and (10) under arbitrary antenna array may be not straightforward.

The proposed new codebook in (6) is identical to the rotated codebook in (4) if the elements in  $\mathbf{G}$  are statistically independent, where the mismatch of rotation matrices  $\|\mathbf{R} - \hat{\mathbf{R}}\|$  is zero, since

$$\begin{aligned} \mathbf{R} &= \mathbf{E}\{\text{vec}(\mathbf{V} \mathbf{G} \mathbf{U}^T) \text{vec}(\mathbf{V} \mathbf{G} \mathbf{U}^T)^H\} = (\mathbf{U} \otimes \mathbf{V}) \mathbf{E}\{\text{vec}\{\mathbf{G}\} \text{vec}\{\mathbf{G}\}^H\} (\mathbf{U} \otimes \mathbf{V})^H \\ &= (\mathbf{U} \otimes \mathbf{V}) \text{diagv}(\boldsymbol{\lambda}) (\mathbf{U} \otimes \mathbf{V})^H = \hat{\mathbf{R}} \end{aligned} \quad (11)$$

where the second equation of (11) is because  $\text{vec}(ABC^T) = (C \otimes A) \text{vec}(B)$ . In this case, the proposed codebook is also asymptotically optimal as the codebook size increases like the rotated codebook in (4).

A sufficient condition for the gain matrix  $\mathbf{G}$  having independent elements can be found similar to [21]. Admittedly, such independency can not be met for general channel conditions, which leads to a nonzero

mismatch of rotation matrices  $\|\mathbf{R} - \hat{\mathbf{R}}\|$ . Then, the proposed codebook becomes suboptimal to the rotated codebook as analyzed in [15]. Yet the performance gap between two codebooks is expected to be small. This is because the dependency between the elements in  $\mathbf{G}$  can be weakened largely after the decorrelation operation by the SVD in (9), similar to what has been observed in [22].

### C. Tucker Decomposition to the Correlation Matrix $\mathbf{R}$

Note that by permutating the elements in  $\mathbf{h}$ , the value of  $\mathbf{V}$ ,  $\mathbf{U}$  and  $\boldsymbol{\lambda}$  given by (9) and (10) is different, and so is the mismatch of rotation matrices  $\|\mathbf{R} - \hat{\mathbf{R}}\|$ . To improve the new rotation matrix, we consider the problem of finding the optimal  $\mathbf{V}$ ,  $\mathbf{U}$  and  $\boldsymbol{\lambda}$  of given dimensionality to minimize the mismatch of rotation matrices under arbitrary form of antenna array, which is modeled as

$$\begin{aligned} \min_{\mathbf{V}, \mathbf{U}, \boldsymbol{\lambda}} \quad & \|\mathbf{R} - (\mathbf{U} \otimes \mathbf{V}) \text{diagv}(\boldsymbol{\lambda}) (\mathbf{U} \otimes \mathbf{V})^H\|^2 \\ \text{s.t.} \quad & \mathbf{V}^H \mathbf{V} = \mathbf{I}_{N_1}, \quad \mathbf{U}^H \mathbf{U} = \mathbf{I}_{N_2}, \quad \boldsymbol{\lambda} \succeq 0 \end{aligned} \quad (12)$$

where  $\mathbf{I}_n$  is the identity matrix of dimension  $n \times n$ , and  $\mathbf{x} \succeq 0$  means each element in  $\mathbf{x}$  is no smaller than 0.

The problem in (12) belongs to a classic approximation problem of Tucker decomposition [23], where  $\boldsymbol{\lambda}$  is known as the core tensor, the columns in  $\mathbf{V}$  and  $\mathbf{U}$  are respectively as tensors. Tucker decomposition is to decompose a higher dimensional matrix into low dimensional factor matrices, and the tensor core encompass all the possible interactions among the low dimensional tensors in the factor matrices. Moreover, Tucker decomposition generalizes many features in the SVD, such as orthogonality, decorrelation and computational tractability, and therefore is desirable for the dimension reduction in this work.

Unfortunately, the problem of Tucker decomposition in (12) is NP-hard in general, and thus there are few efficient algorithms in use. Thus, we turn to find a closed-form solution for the rotation matrix  $\hat{\mathbf{R}}$ .

## IV. CLOSED-FORM SOLUTION FOR $\hat{\mathbf{R}}$

To find a closed-form solution for  $\hat{\mathbf{R}}$  with low complexity, we consider to firstly optimize the  $\mathbf{V}$  and  $\mathbf{U}$  under a structured  $\boldsymbol{\lambda}$  by exploiting the Kronecker product decomposition. Then we optimize  $\boldsymbol{\lambda}$  by using the obtained  $\mathbf{V}$  and  $\mathbf{U}$ .

### A. Optimal $\mathbf{V}$ and $\mathbf{U}$ under a Structured $\boldsymbol{\lambda}$

First, we consider to find the optimal  $\mathbf{V}$  and  $\mathbf{U}$  under a structured  $\boldsymbol{\lambda}$ . Specifically, we impose a Kronecker

product constraint to  $\lambda$  as  $\lambda = \lambda_1 \otimes \lambda_2$ , where  $\lambda_1 \succeq 0$  and  $\lambda_2 \succeq 0$ . Then, we have for problem (12)

$$(U \otimes V) \text{diagv}(\lambda) (U \otimes V)^H = B \otimes C \quad (13)$$

where  $B = U \text{diagv}(\lambda_1) U^H$  and  $C = V \text{diagv}(\lambda_2) V^H$  are positive semi-definite matrices.

By the structured  $\lambda$ , the problem in (12) is reduced to a new problem as

$$\begin{aligned} \min_{B, C} \quad & \|R - B \otimes C\|^2 \\ \text{s.t.} \quad & B \in \mathbb{S}^{N_2 \times N_2} \text{ and } C \in \mathbb{S}^{N_1 \times N_1} \end{aligned} \quad (14)$$

where  $\mathbb{S}^{n \times n}$  is the space of positive semi-definite matrices with the dimensionality of  $n \times n$ .

The new problem in (14) is known as the Kronecker product decomposition in [24], whose optimal solution offers a suboptimal solution of  $V$  and  $U$  to the Tucker decomposition in (12) under a structured  $\lambda$ . According to [24], the optimal solution to the problem in (14) is obtained by rearranging the matrix  $R$ . Specifically, the matrix  $R$  is divided into  $N_2 \times N_2$  blocks as

$$R = \begin{pmatrix} R_{1,1} & \cdots & R_{1,N_2} \\ \vdots & \ddots & \vdots \\ R_{N_2,1} & \cdots & R_{N_2,N_2} \end{pmatrix} \quad (15)$$

where the  $(i, j)$ th block denoted as  $R_{i,j}$  is of dimension  $N_1 \times N_1$ . Then a rearranged matrix of dimension  $N_2^2 \times N_1^2$  is generated as  $\tilde{R} = [\text{vec}(R_{1,1}), \text{vec}(R_{2,1}), \dots, \text{vec}(R_{N_2,N_2})]^T$ .

Denote the largest singular value, the corresponding left and right eigenvectors of the SVD to  $\tilde{R}$  respectively as  $\varrho^2$ ,  $u$  of dimension  $N_2^2 \times 1$  and  $v$  of dimension  $N_1^2 \times 1$ . Then, as shown in [24], the optimal matrices  $B$  of dimension  $N_2 \times N_2$  and  $C$  of dimension  $N_1 \times N_1$  to the problem in (14) are given by

$$\text{vec}(B) = \varrho u \text{ and } \text{vec}(C) = \varrho v \quad (16)$$

Moreover, since  $R$  is symmetric and positive semi-definite,  $B$  and  $C$  are also symmetric and positive semi-definite [24], which can be regarded as the correlation matrices. Thus, the information units  $U$  and  $V$  under a structured  $\lambda$  are obtained by the SVD of the matrix  $B$  and  $C$  respectively as given after (13).

#### B. Optimal $\lambda$ under the Obtained $U$ and $V$



With  $\mathbf{U}$  and  $\mathbf{V}$  obtained in previous subsection, we further optimize the  $\boldsymbol{\lambda}$ . Observing (12), we find that

$$\min_{\boldsymbol{\lambda}} \|\mathbf{R} - (\mathbf{U} \otimes \mathbf{V}) \text{diagv}(\boldsymbol{\lambda}) (\mathbf{U} \otimes \mathbf{V})^H\|^2 = \min_{\boldsymbol{\lambda}} \|(\mathbf{U} \otimes \mathbf{V})^H \mathbf{R} (\mathbf{U} \otimes \mathbf{V}) - \text{diagv}(\boldsymbol{\lambda})\|^2 \quad (17)$$

$$= \min_{\boldsymbol{\lambda}} \|\text{diagm}((\mathbf{U} \otimes \mathbf{V})^H \mathbf{R} (\mathbf{U} \otimes \mathbf{V})) - \text{diagv}(\boldsymbol{\lambda})\|^2 + \|\text{off}((\mathbf{U} \otimes \mathbf{V})^H \mathbf{R} (\mathbf{U} \otimes \mathbf{V}))\|^2 \quad (18)$$

where (17) is because the norm is unitarily invariant, and  $\text{off}(\mathbf{X})$  sets the diagonal in the matrix  $\mathbf{X}$  into zeros.

Therefore, the optimal  $\boldsymbol{\lambda}$  to minimize (18) (and (12)) for the given  $\mathbf{U}$  and  $\mathbf{V}$  is obtained by choosing

$$\text{diagv}(\boldsymbol{\lambda}) = \text{diagm}((\mathbf{U} \otimes \mathbf{V})^H \mathbf{R} (\mathbf{U} \otimes \mathbf{V})) \quad (19)$$

To summarize, the rotation matrix can be constructed as in (7) by using the low-dimensional information unit  $\mathbf{V}$ ,  $\mathbf{U}$  obtained in subsection IV-A and  $\boldsymbol{\lambda}$  obtained in subsection IV-B. The proposed solution is in closed-form and of low complexity, which is thus desirable in practice. The application of the proposed codebook is almost the same as the conventional rotated codebook, differing only in the feedback of rotation matrix. Instead of feeding back  $\mathbf{R}$  directly, the user firstly performs the Tucker decomposition to  $\mathbf{R}$  and feeds back three obtained low-dimensional information units  $\mathbf{V}$ ,  $\mathbf{U}$  and  $\boldsymbol{\lambda}$  to the BS.

## V. SIMULATION RESULTS

In this section, we evaluate the performance of proposed rotated codebook for 3D MIMO systems with planar array by simulations. We consider a multi-user 3D MIMO system, where the BS serves  $K$  single-antenna users simultaneously with zero-forcing beamforming and random user scheduling [5]. The average sum rate is investigated as the performance metric, since a codebook with better quantization performance results in a larger average sum rate. The users have the same receive signal-to-noise ratio (SNR), but different horizontal and vertical clusters in its own 3D MIMO channel.

The channel spatial parameters are set similar in [8]. The horizontal and vertical angles in (1) are modeled respectively as  $\theta_{n,m} = \theta_0 + \theta_n + \delta\theta_{n,m}$  and  $\phi_{n,m} = \phi_0 + \phi_n + \delta\phi_{n,m}$ , where  $\theta_0$  and  $\phi_0$  are the center of horizontal and vertical clusters uniformly distributed respectively in  $(-60^\circ, 60^\circ)$  and  $(-45^\circ, 45^\circ)$ ,  $\theta_n$  and  $\phi_n$  are the horizontal and vertical deviation of the  $n$ th cluster from the center,  $\delta\theta_{n,m}$  and  $\delta\phi_{n,m}$  are small random offsets. We set  $N = 12$  and  $M = 20$ , model  $\theta_n$  and  $\phi_n$  as identically independent distributed (i.i.d.) Gaussian variables with zero mean and a root mean square (RMS) of  $\sigma$ ,  $\delta\theta_{n,m}$  and  $\delta\phi_{n,m}$  as Laplacian variables with a RMS of  $1^\circ$ , and  $g_{n,m}$  are i.i.d. Gaussian variables. Two antenna arrays are studied, i.e., URA and UCCA, where both low and large angular spreads ( $\sigma = 5^\circ$  and  $\sigma = 20^\circ$ ) are considered.

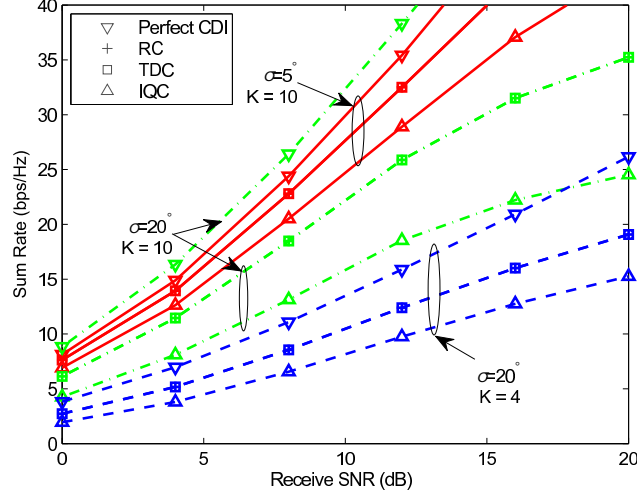


Fig. 2. Average sum rates of 3D MIMO systems using URA for different angular spreads and numbers of users, where  $B = 8$ .

Three codebooks are evaluated in the figures: 1) A  $2B$ -bit rotated codebook (labeled as “RC”) given in (4) quantizes the CDI of 3D MIMO channel directly. 2) A  $2B$ -bit proposed codebook (labeled as “TDC”) given in (6) quantizes the CDI of 3D MIMO channel directly but with reduced dimension in the rotation matrix. 3) The independent quantization (labeled as “IQC”) given in [19] uses two rotated codewords  $\mathbf{c}_h$  and  $\mathbf{c}_v$  to quantize the CDI of horizontal and vertical channel directions respectively in 3D MIMO channel, the feedback CDI at the BS is constructed as  $\mathbf{c}_h \otimes \mathbf{c}_v$ , and the codebook size in each direction is  $B$  bits. The independent quantization avoids the problem of high dimensionality since low-dimensional correlation matrices  $\mathbf{R}_h$  and  $\mathbf{R}_v$  given in (9) are used as the rotation matrices in each direction. Besides, the results with perfect CDI at the BS are provided for comparison. In the simulations, all the correlation matrices are perfectly fed back as in [13]. The codewords  $\mathbf{c}_i$  in different rotated codebooks are obtained from random vector quantization, which are easy to generate but with performance close to the GLP codebooks [5].

Fig. 2 evaluates the average sum rates of three codebooks for 3D MIMO system under URA, where  $N_h = N_v = 8$ ,  $N_1 = N_h$  and  $N_2 = N_v$ . It is shown that under different cases, the proposed codebook in (6) has a performance almost identical to the rotated rotated codebook in (4). This is because URA is particularly natural for the decomposition in (8), (9) and (10) and the elements in  $\mathbf{G}$  are nearly independent as discussed in [21]. Both the two codebooks achieve much of the average sum rates obtained with the perfect CDI at the BS. The performance gain of the proposed codebook over the independent quantization becomes larger as the number of users or the angular spread  $\sigma$  increases. This is because that compared to the independent quantization, the proposed codebook considers not only the dimension reduction, but also the interaction

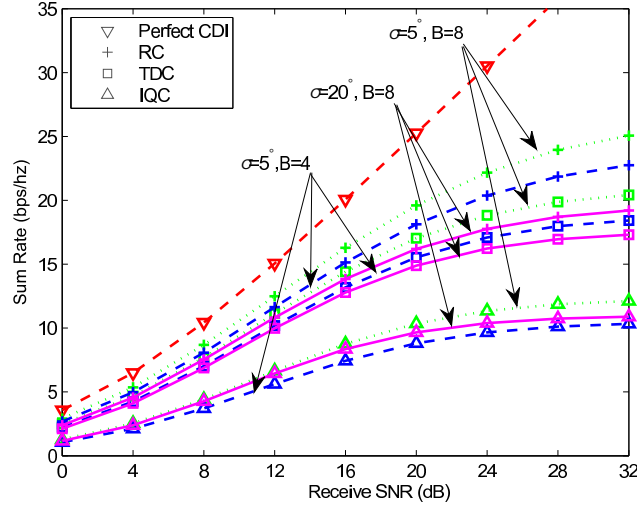


Fig. 3. Average sum rates of 3D MIMO systems using UCCAs with different angle spread and codebook size, where  $K = 4$  and  $\sigma = 5^\circ$ .

between horizontal and vertical directions by introducing the information unit  $\lambda$ .

Fig. 3 evaluates the average sum rates of three codebooks for 3D MIMO system under UCCA, where  $J = L = 8$ ,  $N_1 = J$  and  $N_2 = L$ . The radius of UCCA is set as  $d_j = 0.5j\lambda$  for  $j = 1, \dots, J$ . Similar simulation results can be observed for other settings. As shown in the figure, under different angular spreads  $\sigma$  and codebook sizes, the performance of proposed codebook is slightly inferior to that of rotated codebook. This is because under UCCA, the mismatch of the rotation matrices  $\|\mathbf{R} - \hat{\mathbf{R}}\|$  becomes nonzero. However, the proposed codebook reduces the problem of high dimensionality with acceptable performance loss. Moreover, the proposed codebook outperforms the independent quantization significantly.

## VI. CONCLUSIONS

We have proposed a new rotated codebook for 3D MIMO system under spatial correlated channel by introducing the Tucker decomposition. The closed-form solution is proposed for the new rotation matrix by Tucker decomposition with low computational complexity, and applicable to arbitrary form of antenna arrays. Simulation results have shown the proposed codebook works very well for 3D MIMO systems.

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